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Portfolios in the Ibex 35 before and after the Global Financial Crisis

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ABSTRACT

In this article, we present an analysis of the effectiveness of various portfolio optimization strategies applied to the stocks included in the Spanish Ibex 35 index, for a period of 14 years, from 2001 until 2014. The period under study includes episodes of volatility and instability in financial markets, incorporating the Global Financial Crisis and the European Sovereign Debt Crisis. This implies a challenge in portfolio optimization strategies since the methodologies are restricted to the assignment of positive weights. We have taken for asset allocation the daily returns with an estimation window equal to 1 year and we hold portfolio assets for another year. This article attempts to influence the discussion over whether the naive diversification proves to be an effective strategy as opposed to portfolio optimization models. For that, we evaluate the out-of-sample performance of 15 strategies for asset allocation in the Ibex 35, before and after of the Global Financial Crisis. Our results suggest that a large number of strategies outperform to the 1/N rule and to the Ibex 35 index in terms of return, Sharpe ratio and lower VaR and CVaR. The mean-variance portfolio of Markowitz with short-sale constraints is the only strategy that renders a Sharpe ratio statistically different from Ibex 35 index in the 2001–2007 and 2008–2014 time periods.

KEYWORDS
Finance: portfolio choice; investment decisions; econometrics; minimum-variance portfolios; robust statistics; out-of-sample performance

JEL CLASSIFICATION
C61; G11; C14

I. Introduction

Markowitz (1952, 1959) suggested that a rational investor should choose a portfolio with the lowest risk for a given level of return instead of investing in individual assets, calling these portfolios as efficient. This approach has been the first model of portfolio selection in the literature, which is known as mean-variance of Markowitz. Although the mean-variance methodology has become the central base of the classical finance, leading directly to the development of the Capital Asset Pricing Model (CAPM) by Sharpe (1964), Lintner (1965) and Mossin (1966), the practical application is surrounded by difficulties due to their poor out-of-sample performance since the expected returns are estimated based only on sample information, which results in an estimation error.

A latter approach to addressing the estimation error involves the application of Bayesian techniques, or shrinkage estimators. Jorion (1991) use the Bayesian approach to overcome the weakness of the expected returns estimate only by sample information. More recent approaches are based on the asset pricing model (see Pástor 2000; Pástor and Stambaugh 2000); and the imposition of rules for short-selling constraint (e.g., Frost and Savarino 1988; Chopra 1993; Jagannathan and Ma 2003). Similarly, in the literature have been introduced the minimum-variance portfolios, based on the estimation of the covariance matrix, which is not generally as sensitive to estimation error and provides a better out-of-sample performance (see Chan, Karceski, and Lakonishok 1999; Jagannathan and Ma 2003; among others).

It is also common to use robust optimization techniques to overcome the problems of stochastic programming techniques (see, for example, Quaranta and Zaffaroni 2008; DeMiguel, Garlappi, and Uppal 2009; DeMiguel and Nogales 2009; Harris and Mazibas 2013; Allen et al. 2014a, 2014b; Xing, Hu, and Yang 2014.). Choueifaty and Coignard (2008) and Choueifaty, Froidure, and Reynier (2013) proposed an approach based on the portfolio with the highest ratio of diversification. In addition,

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Supplemental data for this article can be accessed here.

Qian (2005, 2006, 2011) introduced the portfolio with equal contribution to risk, which assigns different weights to assets so that their contribution to the overall volatility of the portfolio is proportional; the properties of this strategy were analysed by Maillard, Roncalli, and Teïletche (2010). These methodologies aim to defend against the possible uncertainty in the parameters of the problem given that these are not exactly known.

In recent years, the interest of the authorities has increased considerably in the measurement of the effects of unexpected losses associated with extreme events in financial markets. This leads directly to improved methodologies for measurement and quantification of risk. In this sense, it is considered that the traditional framework of mean-variance, frequently used in the selection of efficient portfolios, should be revised to introduce more complex risk measures than the simple SD (that is, risk measures based on the quantile). This is the context that explains the choice of Value at Risk (VaR) as synthetic risk measure that can express the market risk of a financial asset or portfolio (JP Morgan 1994). Nevertheless, VaR has been the subject of strong criticism, despite the widespread use in banking supervision, VaR lacks subadditivity so it is not a coherent risk measure for the general distribution of loss, and this goes against the diversification principle (see Artzner et al. 1997, 1999).

Moreover, the absence of convexity of VaR causes considerable difficulties in portfolio selection models based on minimizing the same. Furthermore, the VaR has been criticized for not being able to quantify the so-called 'tail risk'. This has led some researchers to define new risk measures such as Conditional Value at Risk (ES or CVaR) (see Rockafellar and Uryasev 2000, 2002; Pflug 2000; Gaivoronski and Pflug 2005).

There has been a rapid impulse in recent years in the literature about the use of CVaR in portfolio theory. Additionally, the CVaR has the mathematical advantage that can be minimized using linear programming methods. A simple description of the approach to minimize CVaR and CVaR constrained optimization problems can be found in Chekhlov, Uryasev, and Zabarankin (2000). Krokhmal, Palmquist, and Uryasev (2002) compared the CVaR and Conditional Drawdown-at-Risk (CDAR) approaches to minimal risk portfolios in some hedge funds. Agarwal and Naik (2004), and Giamouridis and Vrontos (2007) compared the traditional mean-variance approach with CVaR portfolios built using strategies of hedge funds.

Our objective in this article is to compare the out-of-sample performance of the naive strategy regarding various models for the construction of efficient portfolios. It should be noted that a debate exists in the literature about whether the gains from optimization are reduced by estimation errors or uncertainty in the parameters, which influence in the portfolio optimization process. In this sense, there is no consensus in the literature on whether the naive diversification is more effective than other portfolio strategies (see recent works, such as DeMiguel, Garlappi, and Uppal 2009; Tu and Zhou 2011; Kirby and Ostdiek 2012; and Allen et al. 2014a, 2014b).

For this purpose, we considered a number of optimization models: (a) the classical mean-variance approach (Markowitz 1952, 1959) and the minimum variance approach (Jagannathan and Ma 2003); (b) robust optimization techniques, as the most diversified portfolio(see Choueifaty and Coignard 2008; Choueifaty, Froidure, and Reynier 2013) and the equally weighted risk contributions portfolios (see Qian 2005, 2006, 2011); (c) portfolio optimization based on Conditional Value at Risk, 'CVaR' (Rockafellar and Uryasev 2000, 2002; Alexander and Baptista 2004; Quaranta and Zaffaroni 2008); (d) functional approach based on risk measures such as the 'Maximum draw-down' (MaxDD), the 'Average draw-down' (AvDD), and the 'Conditional draw-down at risk' (CDAR), all proposed by Chekhlov, Uryasev, and Zabarankin (2000, 2005); as well as the Conditional draw-down at risk, 'MinCDaR' (see Chekhlov, Uryasev, and Zabarankin 2005; Kuutan 2007); (e) Young (1998)'s minimax optimization model, based on minimizing risk and optimizing the risk/return ratio; (f) application of Copulae theory to build the minimum tail-dependent portfolio, where the variance–covariance matrix is replaced by lower tail dependence coefficient (see Frahm, Junker, and Schmidt 2005; Fischer and Dörflinger 2006; Schmidt and Stadtmüller 2006); and (g) a defensive approach to systemic risk by beta strategy ('Low Beta'). The beta coefficient (β) is used to assess systemic risk of an asset in the CAPM model (see Sharpe 1964; Lintner 1965;

Mossin 1966), as related volatility of an asset, market, and the correlation between them. To conclude, we impose a short-selling constraint in the models.

Following DeMiguel, Garlappi, and Uppal (2009), it is of paramount importance to compare the results of different methodologies with the 'naive diversification of 1/N', which assigns equal weight to the risky assets. The 1/N strategy has proved as a difficult alternative to beat, demonstrating the practical difficulties to obtain an efficient portfolio (DeMiguel, Garlappi, and Uppal 2009; Allen et al. 2014a.). Therefore, we propose an efficiency analysis of the various methodologies compared with the naive diversification of 1/N and the main Spanish stock index, Ibex 35.

For the evaluation of the out-of-sample performance, we use five criteria. The first one is the Sharpe ratio as a measure of the excess return (Sharpe 1994). To test if the Sharpe ratio of two strategies is statistically different, we obtain the p-value of the difference, using the approach suggested by Jobson and Korkie (1981), after making the correction pointed out in Memmel (2003). Similarly, we calculate the diversification ratio as a measure of the degree of portfolio diversification (Choueifaty and Coignard 2008; Choueifaty, Froidure, and Reynier 2013.); the concentration ratio, which is simply the normalized Herfindahl–Hirschmann index (see Hirschman 1964); The Value at Risk (VaR) as synthetic risk measure that can express the market risk of a financial asset or portfolio, and the expected shortfall (ES or CVaR) as a coherent risk measure that takes into account the 'tail risk'.

As for the data, we use a sample of the daily values of the stocks included into the Ibex 35 index. The Ibex 35 index is the official index of the Spanish Continuous Market. The index is comprised of the 35 most liquid stocks traded on the Continuous market. The prices are adjusted for dividend and these are taken from Datastream. The sample period, running from 1 January 2000 to 31 December 2014, encompasses two episodes of turmoil in financial markets: the Global Financial Crisis, which began in 2008; and the European Sovereign Debt Crisis. The data set is available at the link provided in the supplementary data set section of this paper.

The Spanish stock market has combined from the earlier nineties, when its main stock index Ibex 35 was created; sharp rises with periods of losses. Additionally, the improvement on the technical, operational and organizational systems supporting the market has enabled it to channel large volumes of investment and have made it more transparent, liquid and effective. The pooling of interests has enabled Spain to reach a significant size in the European context and a diversified structure that covers the whole chain of activities in the markets, from trading to settlement. The Ibex 35 stock index has been the subject of numerous studies. For example, Matallin and Nieto (2002) analyse the management of risky assets and mixed risky mutual funds in relation to alternative investment in the Ibex 35. Matallín and Fernández-Izquierdo (2003) examine the extent of the passive timing effect in portfolio management using different portfolios representative of different levels of risk for the Spanish market. Rosillo, De La Fuente, and Brugos (2013) examine the result of the application of technical analysis in the Spanish stock market using different indicators of the quantitative analysis. Fernandez-Perez, Fernández-Rodríguez, and Sosvilla-Rivero (2014) show that the term structure of interest rates has some information content that helps to better forecast the probability of bear markets in the Ibex 35. Finally, Miralles-Quirós, Miralles-Quirós, and Daza-Izquierdo (2015) propose different trading strategies for the Ibex 35, based on the combination of different strategies and on the predictive power of the returns from the opening of the Spanish stock market and the US market.

We used the daily returns with an estimate window equal to one year, 252 days. Therefore, the portfolios have been built for a sample size $N_t = 252$, and the results have been evaluated out of sample for the next period N_{t+1}, (see Table 1). We considered only those stocks that have shown continuity within the index during the period of estimation.[1] We show, in Table 1, the assets number in each period. In the Appendix (Table A2), we report the asset ´considered in each time period.

The rest of the article is organized as follows. In Section 2, we describe the various methodologies

[1]In the Appendix of this article, we include a summary table with the main statistical of the portfolios, and another table with the assets that we consider in each period.

Table 1. Number of assets by time period.

Time period	Number of risky assets
03/01/2000–28/12/2001	21
02/01/2001–30/12/2002	23
02/01/2002–30/12/2003	25
02/01/2003–30/12/2004	26
02/01/2004–30/12/2005	29
03/01/2005–29/12/2006	29
02/01/2006–28/12/2007	30
02/01/2007–30/12/2008	29
02/01/2008–30/12/2009	30
02/01/2009–30/12/2010	31
04/01/2010–30/12/2011	31
03/01/2011–31/12/2012	32
02/01/2012–31/12/2013	33
02/01/2013–31/12/2014	35

used for portfolio construction. In Section 3, we explain the methodology for performance evaluation. In Section 4, we show the results against the Ibex 35 index and the naive strategy of 1/N. In Section 5, we present some concluding remarks.

II. Methodological description

Mean-variance portfolio

The efficient frontier of mean-variance is defined as the set of values $\left(\mu_i, \sigma_i{}^2\right)$ that resolves the following multi-objective optimization problem:

$$\begin{aligned} &\max \boldsymbol{w}\mu, \\ &\min \boldsymbol{w} \sum\nolimits^{w}, \\ &\text{s.t. } \boldsymbol{w}1 = 1, \end{aligned} \tag{1}$$

where $\boldsymbol{w}$ is the $(N \times 1)$ vector of weights and Σ denotes the variance–covariance matrix of asset returns with elements outside the diagonal and σ_{ij} σ_i^2 the ith element of the main diagonal.

Each point on the efficient frontier $\left(\mu_i, \sigma_i^2\right)$ corresponds to an efficient portfolio where the investor gets a maximum return for a given level of risk σ_i. The efficient frontier of mean-variance reflects the relationship between return and risk, introducing the trade-off concept of risk–return in the financial markets. Therefore, it describe the level of return μ_i given a risk exposure σ_i, or seen from a reverse perspective, the lower variability σ_i for a return level μ_i (Markowitz 1952, 1959).

A risk-averse rational investor will make an investment decision on the efficient frontier when the risky asset returns exhibit a multivariate normal distribution or if her utility function is quadratic. The best choice will reflect the investor's willingness to trade off risk against expected return.

To solve efficiently the problem of quadratic optimization with two objectives described above, the problem can be converted into a quadratic optimization problem for different levels of return μ_i (Tsao 2010).

$$\begin{aligned} &\min \boldsymbol{w} \sum\nolimits^{w}, \\ &\text{s.t. } \boldsymbol{w}\mu = \mu_i, \\ &\text{s.t. } \boldsymbol{w}1 = 1, \\ &\text{s.t. } \boldsymbol{w} \geq 0. \end{aligned} \tag{2}$$

The expected return and the variance of the portfolio are $\boldsymbol{w}\mu$, and $\boldsymbol{w}\sum w$, respectively. In this article, we solve the above quadratic optimization problem and establish an expected return μ_i equal to the average return on the assets that are considered in the optimization problem. We have also included a short-selling restriction such that $\boldsymbol{w} \geq 0$.

Minimum-variance portfolio

We use the previous optimization problem to assign the weights $\boldsymbol{w}$ to each asset in the minimum-variance portfolio, but not including the restriction on returns, $\boldsymbol{w}\mu = \mu_i$.

$$\begin{aligned} &\min \boldsymbol{w} \sum \boldsymbol{w}, \\ &\text{s.t. } \boldsymbol{w}1 = 1, \\ &\text{s.t. } \boldsymbol{w} \geq 0. \end{aligned} \tag{3}$$

We obtain the portfolio that provides the minimum variance σ_i^2, given any return μ_i in the efficient frontier of mean-variance. In contrast to the mean-variance portfolio, the minimum variance weight vector does not depend of the expected return on assets (see Jagannathan and Ma 2003, for a study of the properties).

Naive diversification

Several studies confirm the existence of some investors who distribute their wealth through naive diversification strategy. Typically they invest in a few assets alike (see Benartzi and Thaler 2001; Huberman and Jiang 2006). This fact does not prove that the naive diversification is a good strategy, since investors may select a portfolio that is not within the efficient frontier, or she may choose the wrong point in it. Both situations involve a cost, where the second cost is the most important (see Brennan and Torous 1999).

The naive strategy involves a weight distribution $w_j = 1/N$ for all risky assets in the portfolio. This strategy ignores the data and does not involve any estimation or optimization. DeMiguel, Garlappi, and Uppal (2009) suggest that the expected returns are proportional to total risk instead systematic risk.

Equal risk contributed (ERC) portfolio

The portfolios built under the criterion of minimum variance and equally weighted (naive strategy 1/N) are of great interest because they are not based on the expected average returns and therefore they are supposed to be robust. Although the minimum-variance portfolios generally have the disadvantage of a high concentration ratio, it can be limited through diversification (see Qian 2005).

Here is where the equal risk contributed portfolio is located, which assigns different weights to active so that the contribution of these on total portfolio volatility is proportional. Therefore, the diversification is achieved by a weight vector, which is characterized by a distribution of less concentrated portfolio. The ERC portfolio was introduced in the literature by Qian (2005, 2006, 2011) and their properties were analysed by Maillard, Roncalli, and Teïletche (2010).

Maillard, Roncalli, and Teïletche (2010) showed that when it comes to the SD of the portfolio, the ERC solution takes an intermediate position between a minimum-variance portfolio and an equally weighted portfolio. Therefore, the resulting portfolio is similar to a minimum-variance portfolio under additional diversification restrictions.

Let $M(w_i, \ldots, w_N)$ denote a measure of homogeneous risk, which is a function weight w_i of each asset in the portfolio. By Euler's theorem, $M = \alpha \sum_{i=1}^{N} w_i \frac{\partial M}{\partial w_i}$, where α is the degree of homogeneity of M. This leads us to consider the contribution to the risk of asset i to be defined in the form

$$C_i M_{w\in\Omega} = w_i \frac{\partial M_{w\in\Omega}}{\partial w_i} \quad (4)$$

The measure of risk $M_{w\in\Omega}$ can be the SD of the portfolio, the value at risk or the expected shortfall if the degree of homogeneity is one. The portfolio risk is equal to the sum of the risk contributions. If we introduce the formula for the SD portfolio $\sigma(w) = \sqrt{w'\Sigma w}$ to $M_{w\in\Omega}$, then the partial derivatives in the above equation are given by

$$\frac{\partial \sigma(w)}{\partial w_i} = \frac{w_i \sigma_i^2 + \sum_{i\neq j}^{N} w_i \sigma_{ij}}{\sigma(w)} \quad (5)$$

These N partial derivatives are proportional to the ith row of $(\Sigma w)_i$, so the problem for the ERC portfolio with a short-sale constraints and a budget constraint is

$$P_{\text{ERC}} : w_i(\Sigma \mathrm{w})_i = w_j(\Sigma \mathrm{w})_j, \forall i, j \quad (6)$$
$$0 \leq w_i \leq 1$$
$$w'\mathbf{i} = 1$$

where $\mathbf{i}$ is an $(N \times 1)$ vector of ones. The optimal solution of ERC is valid if the value of the objective function is zero, and this only occurs when all contributions imply equal risk. A closed-form solution can only be derived under the assumption that all asset pairs share the same correlation coefficient. Under this assumption, the optimal weights are determined by the ratio of the inverse volatility of the ith asset and the average of the inverse asset volatilities (see Pfaff 2013).

Most diversified portfolio

Choueifaty and Coignard (2008) and Choueifaty, Froidure, and Reynier (2013) studied the theoretical and empirical properties of portfolios when diversification is used as a criterion. To do this, they established a measure for which the degree of diversification for a long portfolio could be evaluated. We define the diversification ratio (DR) to any portfolio P as

$$DR(P) = \frac{w'\sigma}{\sqrt{w' \sum^{w}}} \quad (7)$$

The numerator is the weighted average volatility of the individual assets, divided by the volatility of the portfolio. This relationship has a lower limit of one in the case of a portfolio composed only by an asset. Choueifaty, Froidure, and Reynier (2013) show that the portfolio characterized by a highly concentrated or with strongly correlated asset returns would qualify as being poorly diversified, so that

$$DR(P) = \frac{1}{\sqrt{(\rho + CR) - \rho CR}} \tag{8}$$

where ρ denotes the volatility-weighted average correlation and CR is the volatility-weighted concentration ratio. The DR only depends on the volatility-weighted average correlations in the case of a naive allocation.

Choueifaty, Froidure, and Reynier (2013) established the conditions for the most diversified portfolio by introducing a set of synthetic assets that share the same volatility, such that

$$D(S) = \frac{S'\sum_S}{\sqrt{S'V_sS}} \tag{9}$$

where S is a portfolio composed by synthetic assets, and V_S is the covariance matrix of synthetic assets. If we have to $S'\sum_S = 1$, then to maximize $D(S)$ is equivalent to maximizing $\frac{1}{\sqrt{S'V_sS}}$ under Γ_S restrictions. V_S is equal to the correlation matrix C of initial assets, so that to maximize the diversification ratio is equivalent to minimizing

$$S'CS. \tag{10}$$

Thus, if the assets have the same volatility, the diversification ratio is maximized by minimizing $w'Cw$. Therefore, the objective function coincides with the minimum-variance portfolio, although it is used in the correlation matrix.

The impact of asset volatility is lower in the more diversified portfolio compared with the minimum-variance portfolio (see Pfaff 2013). The weights are retrieved by intermediate vector rescaling weights with SDs of asset returns. The optimal weight vector is determined in two steps: first, an allocation is determined that yields a solution for a least correlated asset mix. This solution is then inversely adjusted by the asset volatilities, and later, the weights of the assets are adjusted inversely by their volatilities.

Minimum tail-dependent portfolio

Minimum tail-dependent portfolio is determined through replacing the variance–covariance matrix by matrix coefficients of lower tail dependence. In that sense, the lower tail of the correlation coefficient measures the dependence of the relationship between the asset returns when these are extremely negative. It is possible to find a scheme with various nonparametric estimators for minimum tail-dependent portfolio in Frahm, Junker, and Schmidt (2005), and Fischer and Dörflinger (2006) and Schmidt and Stadtmüller (2006).

The copulae theory was introduced by Sklar (1959). Sklar's theorem states that there is a C function, called copulae, which establishes the functional relationship between the joint distribution and their univariate marginal distribution functions. Formally, let $x = (x_1, x_2)$ be a two-dimensional random vector with joint distribution function $F(x_1, x_2)$ and marginal distributions $F_i(x_i), i = 1, 2$; there will be a copulae $C(u_1, u_2)$ such that

$$F(x_1, x_2) = P(X_1 < x_1, X_2 < x_2) = C(F_1(x_1), F_2(x_2)). \tag{11}$$

Moreover, Sklar's theorem also provides that if F_i are continuous, then the copulae $C(u_1, u_2)$ is unique. An important feature of copulae is that it allows different degrees of dependency on the tail. The upper tail dependence (λ_U) exists when there is a positive likelihood that positive outliers are given jointly; while the lower tail dependence λ_L, exists when there is a negative likelihood that negative outliers are given jointly (see Boubaker and Sghaier 2013). Thereby, we define the lower tail dependence coefficient as follows:

$$\lambda_L = \lim_{u \to 0} \frac{C(u, u)}{u} \tag{12}$$

This limit can be interpreted as a conditional probability, therefore, the lower tail dependence coefficient is limited in the range $[0, 1]$. The limits are: for an independent copulae $(\lambda_L = 0)$, and for a co-monotonic copulae $(\lambda_L = 1)$. Nonparametric estimators for λ_L are derived from empirical copulae.

For a given sample paired observations N, $(X_1, Y_1), \ldots, (X_N, Y_N)$, with order statistics $X_{(1)} \leq X_{(2)} \ldots \leq X_{(N)}$ and $Y_{(1)} \leq Y_{(2)} \ldots \leq Y_{(N)}$, the empirical copulae is defined as

$$C_N\left(\frac{i}{N}, \frac{j}{N}\right) = \frac{1}{N}\sum_{l=1}^{N} I\left(X_l \leq X_{(i)} \wedge Y_l \leq Y_j\right), \tag{13}$$

with $i,j = 1, \ldots, N$ and I is the indicator function, which has a value of 1 if the condition in parentheses is true. C_N takes a zero value for $i,j = 0$.

In the literature, there are several consistent and asymptotically efficient estimators of λ_L, although this depend on a threshold parameter k, that is the number of statistical order. It is very important to correctly select k in order to estimate the lower tail dependence coefficient; selecting too small a k will result in an inaccurate estimation and a high bias.

For example, the following nonparametric method for estimating of λ_L is derived from a mixture of co-monotonous copulae and independent copulae. The lower tail dependence coefficient is the weight parameter between the two copulae (see Pfaff 2013). So that

$$\lambda_L(N,k) = \frac{\sum_{i=1}^{k}\left(C_N\left(\frac{i}{N},\frac{i}{N}\right) - \left(\frac{i}{N}\right)^2\right)\left(\left(\frac{i}{N}\right) - \left(\frac{i}{N}\right)^2\right)}{\sum_{i=1}^{k}\left(\frac{i}{N} - \left(\frac{i}{N}\right)^2\right)^2} \tag{14}$$

CVaR portfolio

Rockafellar and Uryasev (2000) have advocated for CVaR as a useful measure of risk. Pflug (2000) showed that CVaR is a coherent risk measure with a number of attractive and desirable properties such as monotonicity, translational invariance, positive homogeneity, further CVaR satisfies subadditivity and it is convex.

CVaR is proposed as a method to calculate the market risk arising as a complementary measure to VaR. CVaR is applicable to non-symmetric distributions loss, which takes into account risks beyond the VaR. Furthermore, CVaR accomplishes convexity property with what is possible to identify a global optimum point.

The upper conditional value at risk ($CVaR^+$) is defined as expected losses exceed strictly the VaR; and the lower conditional value at risk ($CVaR^-$) is defined as weakly losses exceeding the VaR (greater or equal losses to VaR). Thus, the conditional value at risk is equal to the weighted average VaR and CVaR^+. CVaR quantifies the excess losses of VaR and acts as an upper bound for the VaR. Therefore, portfolios with low CVaR also have a low VaR. A number of research reports apply CVaR to portfolio optimization problems (see, for example, and Rockafellar and Uryasev 2000, 2002; Andersson et al. 2001; Alexander and Baptista 2004; Rockafellar, Uryasev, and Zabarankin 2006).

In terms of selection of portfolios, CVaR can be represented as a minimization problem of nonlinear programming with an objective function given as

$$\min_{w,v}\frac{1}{na}\sum_{i=1}^{n}\left[\max(0, v - \sum_{j=1}^{m} w_j r_{i,j}\right] \tag{15}$$

where v is the quantile α of the distribution. In the discrete case, Rockafellar and Uryasev (2000) show that it is possible to convert this problem into a linear programming problem by introducing auxiliary variables, so that

$$\min_{w,d,v}\frac{1}{na}\sum_{i=1}^{n} d_i + v \tag{16}$$

$$\sum_{j=1}^{m} w_j r_{i,j} + v \geq -d_i, \forall \in \{1,\ldots,n\}$$

$$\sum_{j=1}^{m} w_j \mu_j = C$$

$$\sum_{j=1}^{m} w_j = 1$$

$$w_j \geq 0, \forall j \in \{1,\ldots,n\}$$
$$d_i \geq 0, \forall i \in \{1,\ldots,n\}$$

where v represents the VaR in the coverage ratio, α and d_i are deviations below the VaR (see Allen et al. 2014b). If the CVaR is minimized, simultaneously, the VAR also will be minimized.

Optimal draw-down portfolios

They are portfolio optimization problems that try to achieve weight solutions with respect to the portfolio's draw-down. This kind of optimization was proposed by Chekhlov, Uryasev, and Zabarankin (2000, 2005). The task of finding optimal portfolio allocations with respect to draw-down is of considerable interest to asset managers, as it is possible to avoid, somehow, large withdrawals and/or loss of revenue management.

The draw-down of a portfolio at time t is defined as the difference between the maximum uncompounded portfolio value prior to t and its value at t. Formally, denote by $W(w,t) = y_t' w$ the uncompounded portfolio value at time t, with w the portfolio weights for the N assets included in it, and y_t is the accumulated returns. Then the draw-down, $\mathbb{D}(w,t)$, is defined as

$$\mathbb{D}(w,t) = \max_{0\le\tau\le 1}\{W(w,\tau)\} - W(w,t). \qquad (17)$$

Chekhlov, Uryasev, and Zabarankin (2000) deducted three functional measures of risk: maximum draw-down (MaxDD), average draw-down (AvDD) and conditional draw-down at risk (CDaR). CDaR is dependent on the chosen confidence level α. CDaR is a measure of functional risk and not a risk measure as in the case of CVaR. The limiting cases of this family of risk functions are MaxDD and AvDD,

$$\mathrm{CDaR}(w)_\alpha = \min_\zeta\left\{\zeta\frac{1}{(1-\alpha)T}\int_T^0\left[\mathbb{D}(w,t)-\zeta\right]^+\mathrm{d}t\right\} \qquad (18)$$

where ζ is a threshold value for the draw-downs, so that only $(1-\alpha)T$ observations exceed this value.

For $\alpha \to 1$, CDaR approaches the maximum draw-down: $\mathrm{CDaR}(w)_{\alpha\to 1} = \mathrm{MaxDD}(w) = \max_{0\le t\le T}\{\mathbb{D}(w,t)\}$. The AvDD result for $\alpha = 0$ is $\mathrm{CDaR}(w)_{\alpha=0} = \mathrm{AvDD}(w) = (1/T)\int_T^0 \mathbb{D}(w,t)dt$.

The portfolio optimization is expressed in discrete terms and the objective is defined as maximizing the annualized average return of the portfolio (see Pfaff 2013),

$$R(w) = \frac{1}{dC}y'_T w \qquad (19)$$

where d is the number of years in the time interval $[0,T]$. In short, we consider the three functional risk measures, MaxDD, AvDD and CDaR, proposed by Chekhlov, Uryasev, and Zabarankin (2000, 2005). Further, we consider the minimization of CDaR:

$$\begin{aligned}
P_{\mathrm{MaxDD}} &= \arg\max_{w,u}\ R(w) = \frac{1}{dC}y_T w \qquad (20)\\
&u_k - y'_k w \le v_1 C\\
&u_k \ge y'_k w\\
&u_k \ge u_{k-1}\\
&u_0 = 0
\end{aligned}$$

where $\boldsymbol{u}$ denotes a $(T+1\times 1)$ vector of slack variables in the program formulation, in effect, the maximum portfolio values are up to time period k with $1\le k\le T$. When the portfolio is optimized with regard to limiting of the average draw-down, only the first set of inequality constraints needs to be replaced with the discrete analogue of the mean draw-down expressed in continuous time as indicated above (see Pfaff 2013), which result in

$$\begin{aligned}
P_{\mathrm{AVDD}} &= \arg\max_{w,u} R(w) = \frac{1}{dC}y'_T w, \qquad (21)\\
&\frac{1}{T}\sum_{k=1}^{T}\left(u_k - y'_k w\right) \le v_2 C\\
&u_k \ge y'_k w\\
&u_k \ge u_{k-1}\\
&u_0 = 0.
\end{aligned}$$

For the CDaR linear programming problem it is necessary to introduce two additional auxiliary variables, the threshold draw-down value ζ dependent on the confidence level α, and the $(T\times 1)$ vector $\mathbf{z}$, representing the weak threshold exceedances; so that

$$\begin{aligned}
P_{\mathrm{CDaR}} &= \arg\max_{w,u,z,\zeta} R(w) = \frac{1}{dC}y'_T w \qquad (22)\\
&\zeta + \frac{1}{(1-\alpha)T}\sum_{k=1}^{T} z_k \le v_3 C,\\
&z_k \ge u_k - y'_k w - \zeta,\\
&z_k \ge 0,\\
&u_k \ge y' k w,\\
&u_k \ge u_{k-1},\\
&u_0 = 0.
\end{aligned}$$

The minimization of CDaR (see Chekhlov, Uryasev, and Zabarankin 2005; Kuutan 2007) can be obtained similarly to the conditional value at risk (CVaR) through linear optimization, but we have to introduce auxiliary variables:

$$\begin{aligned}
P_{\mathrm{MinCDaR}} &= \mathrm{argmin} y + \frac{1}{(1-\alpha)T}\sum_{t=1}^{T} z_k, \qquad (23)\\
&z_k \ge u_k - r_p(w,t) - y,\\
&z_k \ge 0,\\
&u_k \ge r_p(w,t),\\
&u_k \ge u_{k-1},
\end{aligned}$$

where y is the threshold value of the accumulative distribution function $D(w,t)$, and z_k, u_k are auxiliary variables.

The limitations $u_k \ge r_p(w,t)$, and $u_k \ge u_{k-1}$ replace linearly the higher value of the portfolio till the moment $t:\max\{r_p(w,t)\}$. The first constraint ensures that u_k is always higher or at least equal to the portfolio accumulated return in the moment k, and the second constraint ensures that u_k is always higher or at least equal to the previous value (see Kuutan 2007). Before of the optimization process, y is a free variable, after the optimization process it is the $CDaR_\alpha$ for the MinCDaR portfolio. Thus, if we minimize the function $H_\alpha(w, y)$, we simultaneously obtain both values (see Albina Unger 2014).

Minimum tail-dependent portfolio based in Clayton copulae and low beta strategy

The minimum tail-dependent is derived from a Clayton copulae. The Clayton copulae belongs to the family of Archimedean copulae; its one of the most used in the literature (see Clayton 1978). An Archimedean generator, or generator, is a continuous decreasing function $\psi : [0, \infty] \to [0, 1]$, which complies with $\psi(0) = 1, \psi(\infty) := \lim_{t \to \infty} \psi(t) = 0$, and that is strictly decreasing on $[0, \inf\{t : \psi(t) = 0\}]$. The set of all functions is denoted by Ψ.

An Archimedean generator $\psi \in \Psi$ is called strict if $\psi(t) < 0$ for all $t \in [0, \infty]$. A d-dimensional copulae C is called Archimedean (see Hofert and Scherer 2011) if it allows the representation

$$C(u) = C(u; \psi) := \psi\left(\psi^{-1}(u_1) + \ldots + \psi^{-1}(u_d)\right), u \in I^d \tag{24}$$

for some $\psi \in \Psi$ with inverse $\psi^{-1} : [0, 1] \to [0, \infty]$, where $\psi^{-1}(0) := \inf\{\mathrm{t} : \psi(\mathrm{t}) = 0\}$. There are different notations for Archimedean copulae. A bivariate Clayton copulae can be presented so that

$$C(u_1, u_2) = \psi^{-1}(\psi(u_1) + \psi(u_2)) = \left(u_1^{-\delta} + u_2^{-\delta} - 1\right)^{1/\delta} \tag{25}$$

The Clayton copula has the minimum tail-dependence. The coefficient is calculated according to $\lambda_l = 2^{-1/\delta}$. For the bivariate Clayton copulae, the following simplifications are given:

$$\hat{\delta} = \frac{2\hat{\rho}\tau}{1 - \hat{\rho}\tau} \tag{26}$$

$$\hat{\theta} = \frac{1}{1 - \hat{\rho}\tau} \tag{27}$$

where $\hat{\rho}\tau$ is the empirical Kendall rank correlation (see, for example, Genest and Favre 2007).

In addition, we implemented the strategy of lower beta coefficient ('Low Beta'); beta (β) is the coefficient used to evaluate systemic risk of an asset in the CAPM model (see Sharpe 1964; Lintner 1965; Mossin 1966); it relates the volatility of an asset, market and the correlation between them.

We select assets whose volatility is less than the reference market, in absolute terms, for the construction of the beta portfolio. The process to build the portfolio can be summarized so that, we get the beta coefficients of each asset such that

$$\beta_i = \frac{\mathrm{Cov}(R_i, R_b)}{\sigma_b^2} \tag{28}$$

where the numerator represents the covariance between assets i and the market b, and the denominator is the variance of the market.

Then, we select those assets whose β coefficients and coefficients of tail dependence are below their respective medians. Finally, we get the weights by applying an inverse logarithmic scale (this application can be seen in Pfaff 2013). Both strategies are referred to as defensive relative to the market (benchmark), as they are aimed at minimizing systemic risk.

Minimax portfolios based on risk minimization and optimization of the risk/return ratio

The Minimax model (see Young 1998) aims to minimize the maximum expected loss, thus it is a very conservative criterion. Formally, when it is applied to the selection of portfolios, given N assets and t periods, the model can be presented as a linear programming problem, such that

$$\min_{M_{p,w}} M_p \tag{29}$$

$$M_p - \sum_{j=1}^{m} w_j r_{i,j} \leq 0, \forall i = \{1, \ldots, n\}$$

$$\sum_{j=1}^{m} w_j \mu_J = C$$

$$\sum_{j=1}^{m} w_j = 1$$

$$w_j \geq 0, \forall j \in \{1, \ldots, n\}$$

where M_p is the target value to minimize, which represents the maximum loss of the portfolio given a weight vector w, C is a certain minimum level of return, and μ denote the forecast for the returns vector of m values. In principle, Minimax is consistent with the theory of expected utility in the limit based on an investor who is very risk averse. Furthermore, the minimax model is a good approximation to the mean-variance model when the asset returns follow a multivariate normal distribution.

If we draw the portfolios set for different levels of C (using an equality rather than inequality), it is possible to generate the frontier portfolio from which the optimal risk portfolio can be chosen. It is possible to estimate the optimal risk/return using fractional programming as it is described in Charnes and Cooper (1962), and more recently in Stoyanov, Rachev, and Fabozzi (2007). The Minimax linear programming problem can be reformulated, so that

$$\min_{M_{p,wb}} M_p \tag{30}$$

$$M_p - \sum_{j=1}^{m} w_j r_{i,j} \leq 0, \forall i = \{1, \ldots, n\}$$

$$\sum_{j=1}^{m} w_j \mu_J = 1$$

$$\sum_{j=1}^{m} w_j = b$$

$$b \geq 0$$

where b is the multiplier coefficient added to the optimization problem as a result of transformation of the risk/return problem. More details can be found in Charnes and Cooper (1962) for LP (linear programing), and in Dinkelbach (1967) for NLP (nonlinear programing).

In summary, we use two types of optimization: the first optimization is based on risk minimization, and the second optimization is based on the risk/return ratio. Table 2 provides a list of asset-allocation models considered.

III. Methodology for evaluating performance

We take the out-of-sample daily returns for one year, and we assign the weights determined by the portfolio optimization process to each asset i. We consider five measures for statistical comparison between the portfolio strategies: Value at Risk (VaR), Conditional Value at Risk (CVaR), Sharpe ratio, diversification ratio and concentration ratio. Results are provided for three time periods, 2001–2014, 2001–2007 and 2008–2014.

Table 2. List of asset-allocation models considered.

Methodology	Model	Abbreviation
1. Naive Diversification	• Naive strategy of $1/N$	1/N
2. Classic	• Mean-variance portfolio	M-V
3. Robust Portfolios	• Minimum-variance portfolio	GMV
	• Most diversified portfolio	MDP
	• Equal risk contributed portfolio	ERC
	• Minimum tail-dependent portfolio	MTD
4. CVaR Portfolio	• Conditional value at risk portfolio	CVaR
5. Draw-down Portfolios	• Maximum draw-down portfolio	MaxDD
	• Average draw-down portfolio	AvDD
	• Conditional draw-down at risk (95%)	CDaR95
	• Minimum conditional draw-down at risk (95%)	MinCDaR95
6. Minimax Portfolios	• Minimax based on risk minimization	R-Minimax
	• Minimax based on the risk/return ratio	O-Minimax
7. Defensive Portfolios	• Minimum tail-dependent with Clayton copulae	Clayton (MTD)
	• Low beta portfolio	Beta

Value at risk and conditional value at risk

Value at Risk (VaR) is a measure of synthetic risk that can express the market risk of a financial asset or portfolio. In general terms, VaR is the maximum potential loss that a financial asset may suffer with a certain probability for a certain period of tenure. JP Morgan tried to establish a market standard by RiskMetrics in 1994 (JP Morgan 1994).

For a confidence level $\alpha \in (0, 1)$, VaR is defined as the smallest number l such that the probability of loss L is not greater than $1 - \alpha$ for greater losses that l. This value corresponds to the quantiles of loss distribution, and it can be formally expressed as

$$\begin{aligned} \mathrm{VaR}_\alpha &= \inf\{l \in \mathbb{R} : \mathrm{P}(L>l) \leq 1 - \alpha\} \\ &= \inf\{l \in \mathbb{R} : F_L(l) \geq \alpha\} \end{aligned} \tag{31}$$

where F_L is the distribution function of the losses (see Pfaff 2013).

The expected shortfall risk measure (ES or CVaR) arises due to deficiencies that VaR shows. CVaR was introduced by Artzner et al. (1997, 1999); Rockafellar and Uryasev (2002) showed that CVaR is a consistent measure of risk and may also take into consideration the ‘tail risk’.

CVaR is defined for a type I error α as

$$ES_\alpha = \frac{1}{1-\alpha} \int_\alpha^1 q_u(F_L)\mathrm{d}u \tag{32}$$

where $q_u(F_L)$ is the quantile function of loss distribution F_L. Therefore ES can be expressed in VaR terms such that

$$ES_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_u(L)\mathrm{d}u \tag{33}$$

ES can be interpreted as the VaR average in the range $(1-\alpha, 1)$.

Sharpe ratio

We calculate the out-of-sample annualized Sharpe ratio for each strategy z. Sharpe ratio is defined as the sample mean of out-of-sample excess returns over the risk-free asset $\hat{\mu}_z$, divided by their sample SD $\hat{\sigma}_z$, such that

$$Sharpe\,R = \frac{\hat{\mu}_z}{\hat{\sigma}_z}. \tag{34}$$

To test the statistical independence of the Sharpe ratios for each strategy with respect to benchmark, we calculate the p-value of the difference, using the approach suggested by Jobson and Korkie (1981) after making the correction pointed out in Memmel (2003), and recently applied in DeMiguel, Garlappi, and Uppal (2009). So that, given two portfolios a and b, with mean $\hat{\mu}_a, \hat{\mu}_b$, variance $\hat{\sigma}_a, \hat{\sigma}_b$, and covariance $\hat{\sigma}_{a,b}$ about a sample of size N, its checked by the test statistic $\hat{z}_{JK}$, the null hypothesis that $H_0 : \hat{\mu}_a/\hat{\sigma}_a - \hat{\mu}_b/\hat{\sigma}_b = 0$. This test is based on the assumption that income is distributed independently and identically (IID) in time following a normal distribution, (see Jobson and Korkie 1981; Memmel 2003).

Diversification ratio and concentration ratio

We define diversification ratio (DR) to any portfolio P as follows:

$$DR(P) = \frac{w'\sigma}{\sqrt{w'\sum^{w}}}. \tag{7}$$

The numerator is the weighted average volatility of the single assets, divided by the portfolio volatility (portfolio SD). From the above equation we derive the following expression:

$$DR(P) = \frac{1}{\sqrt{(\rho + CR) - \rho CR}} \tag{8}$$

where ρ denotes the volatility-weighted average correlation and CR is the volatility-weighted concentration ratio. The parameter ρ is defined as

$$\rho = \frac{\sum_{i\neq j}^{N}\left(w_i\sigma_i w_j\sigma_j\right)\rho_{ij}}{\sum_{i\neq j}^{N}\left(w_i\sigma_i w_j\sigma_j\right)} \tag{35}$$

The concentration ratio (CR) is the normalized Herfindahl–Hirschmann index (see Hirschman 1964):

$$CR(P) = \frac{\sum_{i=1}^{N}\left(w_i\sigma_i\right)^2}{\sum_{i=1}^{N}\left(w_i\sigma_i\right)^2} \tag{36}$$

IV. Results

In this section, we compare the out-of-sample results obtained for the various portfolio strategies. For that, we show the results of the five measures for statistical comparison between the portfolio strategies, contained in the previous section. The portfolio strategies results are compared with the Ibex 35 index and the naive strategy of 1/N. Table A1 in the Appendix provides descriptive statistics sumarising the behaviour during different time perios of the Ibex 35 index and the 15 main models consider in this article.

We take the out-of-sample daily returns for one year, and we assign the weights determined by the portfolio optimization process to each asset i considered, so that we build the portfolio and analyse it for next year. Therefore, we build portfolios with the daily returns series of N_t period and they are tested for the following period, N_{t+1}, for $t = 2000, 2001, \ldots, 2013$. We have built 14 portfolios for methodological framework, although the results are aggregated by time periods: 2001–2014, 2001–2007 and 2008–2014.

In the first and second columns of Tables 3–5, we present the total return (Total Return) and the annualized return (Annual Return) of each strategy for the time periods 2001–2014 (Table 3), 2001–2007 (Table 4) and 2008–2014 (Table 5). The value at risk and the conditional value at risk (1 day) appear in the third and fourth columns, respectively. The Sharpe ratio and the p-value of each strategy, including the Ibex 35 index, are shown in the fifth column. We also include the p-value of the difference for each strategy with respect to Ibex 35 index. In the last two columns, six and seven, we report the diversification and concentration ratios, respectively.

Table 3. Summary of main results, 2001–2014 time period.

Portfolio	Total Return	Annual Return	VaR 95% 1 day	CVaR 95% 1 day	Annualized Sharpe ratio (*p*-value)	Diversification ratio	Concentration ratio
Ibex 35	13.21%	0.89%	2.525	3.170	0.0365 (1.000)	–	–
1/N	46.80%	2.78%	2.165	2.720	0.1321 (0.493)	1.5648	**0.0400**
M-V	**276.37%**	**9.93%**	**1.778**	**2.240**	**0.5663** (0.005)***	1.5989	0.1551
GMV	**244.31%**	**9.23%**	**1.689**	**2.129**	**0.5544** (0.009)***	1.6102	0.1513
MDP	179.21%	7.61%	**1.742**	**2.194**	**0.4440** (0.038)**	**1.7153**	0.1113
ERC	81.18%	4.34%	1.988	2.499	0.2239 (0.182)	**1.6164**	**0.0391**
MTD	117.47%	5.71%	1.836	2.310	0.3176 (0.092)*	**1.6326**	**0.0973**
CVaR	154.00%	6.88%	**1.786**	**2.248**	0.3896 (0.096)*	1.5100	0.2059
MaxDD	159.24%	7.04%	2.383	2.997	0.3041 (0.261)	1.3464	0.3830
AvDD	193.39%	7.99%	3.170	3.988	0.2585 (0.338)	1.0788	0.8401
CDaR95	167.64%	7.29%	2.437	3.066	0.3067 (0.251)	1.2765	0.4879
MinCDaR95	**291.71%**	**10.24%**	1.937	2.441	**0.5375** (0.030)**	1.4157	0.2901
R-Minimax	105.82%	5.29%	2.006	2.523	0.2707 (0.239)	1.4725	0.2141
O-Minimax	**247.09%**	**9.3%**	2.188	2.754	0.4317 (0.096)*	1.2563	0.5223
Clayton (MTD)	184.78%	7.76%	1.852	2.332	0.4257 (0.041)**	**1.6932**	**0.0954**
Beta	**246.43%**	**9.28%**	**1.705**	**2.148**	**0.5515** (0.008)***	**1.6783**	**0.1008**

Table 4. Summary of main results, 2001–2007 time period.

Portfolios	Total Return	Annual Return	VaR 95% 1 day	CVaR 95% 1 day	Annualized Sharpe ratio (*p*-value)	Diversification ratio	Concentration ratio
Ibex 35	67.2%	7.62%	2.038	2.566	0.3805 (1.000)	–	–
1/N	139.01%	13.26%	1.579	1.994	0.8410 (0.045)**	1.7043	**0.0416**
M-V	**160.57%**	**14.64%**	**1.302**	**1.648**	**1.1173** (0.015)**	1.7405	0.1131
GMV	**173.18%**	**15.44%**	**1.247**	**1.580**	**1.2237** (0.008)***	**1.7656**	0.1061
MDP	126.14%	12.36%	**1.333**	**1.684**	0.9267 (0.095)*	**1.8935**	**0.0904**
ERC	143.36%	13.55%	1.430	1.807	0.9458 (0.024)**	**1.7764**	**0.0419**
MTD	129.88%	12.59%	1.359	1.718	0.9280 (0.041)**	1.7629	**0.0794**
CVaR	138.64%	13.23%	**1.326**	**1.676**	**0.994** (0.074)*	1.6635	0.1581
MaxDD	124.13%	12.22%	1.708	2.154	0.7201 (0.448)	1.4408	0.3433
AvDD	84.53%	9.15%	2.795	3.518	0.3332 (0.978)	1.1311	0.7649
CDaR95	133.05%	12.85%	1.783	2.250	0.7248 (0.418)	1.4006	0.3882
MinCDaR95	**146.73%**	**13.77%**	1.623	2.049	0.8500 (0.216)	1.5151	0.2409
R-Minimax	127.53%	12.46%	1.461	1.846	0.8541 (0.127)	1.5664	0.2012
O-Minimax	67.69%	7.66%	1.882	2.369	0.4142 (0.936)	1.3620	0.4025
Clayton (MTD)	**168.32%**	**15.14%**	1.381	1.747	**1.0895** (0.051)*	**1.8634**	0.0989
Beta	**167.13%**	**15.07%**	**1.249**	**1.581**	**1.1946** (0.020)**	**1.8572**	**0.0928**

Results for the period comprises between 2001 and 2007. In parenthesis, the *p*-value corresponding to the $\hat{z}_{JK}$ test; The asterisks show the significance of the tests: weak significance (*), moderate significance (**), strong significance (***).Bold values indicates the five best-performing portfolios according to each metric.

Table 5. Summary of main results, 2008–2014 time period.

Portfolios	Total Return	Annual Return	VaR 95% 1 day	CVaR 95% 1 day	Annualized Sharpe ratio (*p*-value)	Diversification ratio	Concentration ratio
Ibex 35	−31.48%	−5.26%	2.879	3.609	−0.1873 (1.000)	–	–
1/N	−38.58%	−6.73%	2.599	3.256	−0.2663 (0.481)	1.4252	**0.0383**
M-V	**44.44%**	**5.39%**	**2.126**	**2.674**	**0.2559** (0.065)*	**1.4573**	0.1971
GMV	26.03%	3.36%	**2.017**	**2.534**	0.1687 (0.171)	1.4548	0.1964
MDP	23.47%	3.06%	**2.054**	**2.580**	0.1508 (0.171)	**1.5370**	0.1322
ERC	−25.55%	−4.13%	2.398	3.006	−0.1765 (0.938)	1.4564	**0.0362**
MTD	−5.40%	−0.79%	2.192	2.751	−0.0368 (0.508)	**1.5023**	0.1153
CVaR	6.43%	0.89%	**2.154**	**2.705**	0.0422 (0.407)	1.3565	0.2538
MaxDD	15.66%	2.10%	2.846	3.575	0.0749 (0.335)	1.2521	0.4227
AvDD	**58.99%**	**6,85%**	3.436	4.321	**0.2010** (0.201)	1.0264	0.9153
CDaR95	14.84%	2.00%	2.900	3.643	0.0699 (0.357)	1.1524	0.5876
MinCDaR95	**58.76%**	**6.83%**	2.171	2.731	**0.3165** (0.081)*	1.3162	0.3392
R-Minimax	−9.54%	−1.42%	2.401	3.012	−0.0605 (0.637)	1.3786	0.2270
O-Minimax	**106.98%**	**10,95%**	2.424	3.053	**0.4522** (0.049)**	1.1506	0.6421
Clayton (MTD)	6.13%	0.85%	2.213	2.778	0.0393 (0.292)	**1.5229**	**0.0919**
Beta	**29.69%**	**3.78%**	**2.045**	**2.570**	**0.1872** (0.104)	**1.4993**	**0.1089**

Results for the period comprises between 2008 and 2014. In parenthesis, the *p*-value corresponding to the $\hat{z}_{JK}$ test; The asterisks show the significance of the tests: weak significance (*), moderate significance (**), strong significance (***).Bold values indicates the five best-performing portfolios according to each metric.

Out-of-sample performance: 2001–2014

Five strategies have an annual return equal or greater than 9%, compared with the Ibex 35 index, that does not exceed 1% by year. This can be seen more intuitive when considering the total return since 2001. The MinCDaR95 portfolio achieved a total return equal to 291.71%, followed by the MV, O-Minimax, Beta and GMV portfolios, with a total return greater than 240%. During the same period, the Ibex 35 index increased 13.21%, being followed in terms of lower returns by two portfolios based on the naive diversification, the 1/N and ERC strategies, with a total return of 46.80% and 81.18%, respectively. All strategies have a lower VaR and CVaR than the Ibex 35 index (2.52 and 3.17), except the AvDD portfolio. The GMV portfolio stands out as the portfolio with lower VaR and CVaR (1.77 and 2.12, respectively).

Four strategies achieve an annualized Sharpe ratio of 0.5. The MV portfolio emerges with a Sharpe ratio equal to 0.566, followed by the GMV, the Beta and the MinCDaR95 portfolios with 0.554, 0.551 and 0.537, respectively. Considering the p-value, the above-mentioned strategies turn out to be moderate or very significant, that is, their Sharpe ratios do differ statistically with respect to the Ibex 35 index. The 1/N and ERC strategies render Sharpe ratios well below the MV and GMV portfolios, indeed, if we exclude the Ibex 35 index, the 1/N and ERC portfolios have the lowest Sharpe ratios.

The MDP, Clayton (MTD) and Beta strategies present the highest ratios of diversification, the first one standing out with a ratio of 1.71. The MDP portfolio is among the strategies with a higher Sharpe ratio, showing the possibility to obtain a high Sharpe ratio and at the same time a considerable diversification ratio. In addition, four strategies exceed the diversification ratio of the 1/N strategy, such as the MTD, the ERC, the GMV and the M-V portfolios. All these with a diversification ratio between the values of 1.71 (MDP portfolio) and 1.59 (MV portfolio).

Results for the period comprises between 2001 and 2014. In parenthesis, the p-value corresponding to the $\hat{z}_{JK}$ test; The asterisks show the significance of the tests: weak significance (*), moderate significance (**), strong significance (***). Bold values indicates the five best-performing portfolios according to each metric.

The concentration ratio rewards the largest share of assets in the portfolio, so the portfolios based on the naive diversification have the lowest concentration ratios (the ERC portfolio with 0.039 and the 1/N portfolio with 0.04), followed by two portfolios based on the lower tail dependence: the Clayton (MTD) and MTD portfolios, with 0.095 and 0.097, respectively. The concentration ratio can be related to the cost of building the portfolio because the concentration ratio decreases when the number of assets increases in the portfolio.

In Figure 1, we show the poor performance of the Ibex 35 index and the naive strategy of 1/N with respect to the other four methodologies considered (with a Sharpe ratio greater than 0.5). The differences between the Ibex 35 index and the strategies are relevant from 2002, although the greatest divergence is reached in 2014. At the end of the time period under study (the year 2014), the Ibex 35 index registered a total return of 13.21% in contrast to the rest of strategies, which achieved a minimum total return of 240%, except for the naive strategy of 1/N (with a total return of 46.80%).

Base 100 in February 1st 2001. We represent the accumulated wealth of an investor who invested 100 currency units on February 1st 2001. We include the Ibex 35 index, the 1/N portfolio, the M-V portfolio, the GMV portfolio, the MinCDaR95 portfolio and the Beta portfolio.

The accumulated wealth generated by the naive strategy of 1/N was similar to other portfolios during the 2001–2007 period. However, the naive strategy performance is very similar to that the of the Ibex 35 index in the 2008–2014 period. In short, this fact causes that the return of the 1/N portfolio in the 2001–2014 time period is 46.8%, clearly surpassed at least by nine strategies, among which the MinCDaR95 and the M-V portfolios stand out.

In 2008 the world economy faced its most dangerous crisis since the Great Depression of the 1930s, being known as the Global Financial Crisis. Share prices plunged throughout the world – the Dow Jones Industrial Average in the United States lost 33.8% of its value in 2008 – and by the end of the year, a deep recession had enveloped most of the globe. Therefore, we examine separately the out-of-sample performance for the 2001–2007 and 2008–2014 time periods.

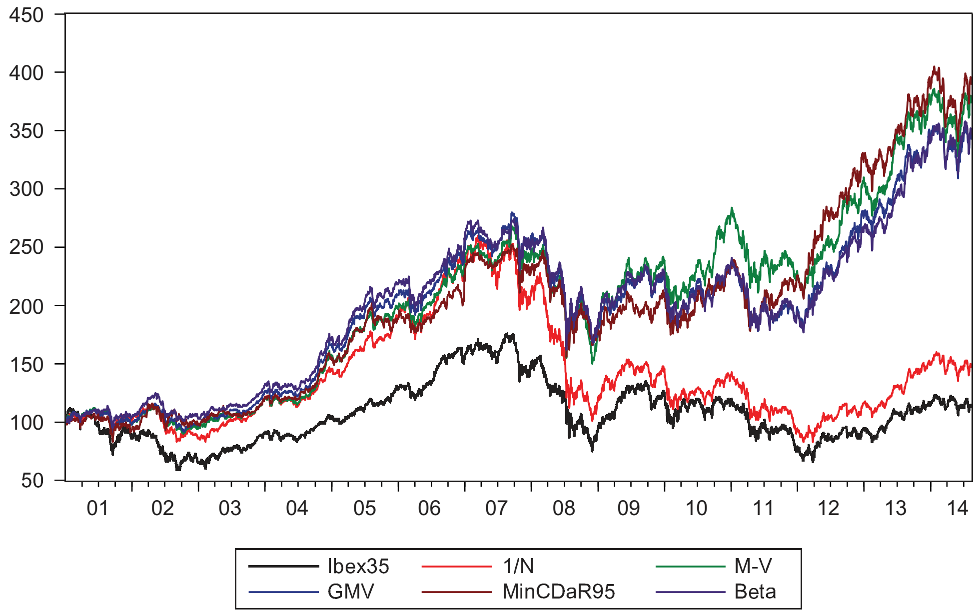

Figure 1. Accumulated wealth, 2001–2014 period.

Out-of-sample performance: 2001–2007

The GMV, the Clayton (MTD) and the Beta portfolios achieve an annual return equal or greater than 15%. In total, there are six strategies that outperform the naive strategy of 1/N, whose annual return is 13.26%. In line with the results for the entire sample period (2001–2014), the return of the Ibex 35 index is exceeded by all portfolio strategies. Thus, the MV, GMV, Clayton (MTD) and Beta portfolios get a total return equal to or greater than 160%. The GMV portfolio achieves the higher performance with a total return of 173.18%, in contrast with the Ibex 35 index, with a total return of 67.20%.

Except for the AvDD portfolio, the rest of strategies dominate the Ibex 35 index (VaR of 2.03 and CVaR of 2.566) in terms of Value at Risk and Conditional Value at Risk at one day. The GMV portfolio is the strategy with the lowest VaR and CVaR (1.24 and 1.64, respectively) followed for the Beta, the M-V and the CVaR portfolios. In advance, one would expect that the CVaR portfolio would obtain a lower VaR and CVaR than other portfolios; however, this strategy is overcome by the three portfolios listed previously. In addition, there are nine portfolios that show a smaller VaR and CVaR than that is obtained for the strategy of 1/N (1.57 and 1.99, respectively).

The GMV, M-V, Beta, Clayton (MTD) and CVaR portfolios achieve an annualized Sharpe ratio near or above 1, in contrast to the Sharpe ratio obtained for the Ibex 35 index (0.380). The GMV portfolio stands out with a Sharpe ratio equal to 1.223, followed by the Beta, M-V and Clayton (MTD) portfolios with 1.194, 1.117 and 1.089, respectively. In addition, there are nine portfolios whose Sharpe ratios differ statistically from the Ibex 35, that is, all portfolio strategies except those based on the minimax model and the conditional drawdown-at-risk approaches.

The MDP, Clayton (MTD) and Beta strategies have the highest diversification ratios, greater than 1.85. On the other hand, the AvDD, O-Minimax and CDaR95 strategies have low diversification ratios, none greater than 1.4. The GMV and M-V portfolios exceed the diversification ratio of the 1/N strategy. Finally, seven portfolios are able to overcome the strategy of 1/N in terms of diversification ratio.

Again, portfolios based on the naive diversification are those that have a lower concentration ratio, slightly lower for the 1/N portfolio (0.041). In contrast, the AvDD, O-Minimax, CDaR95 and MaxDD strategies are highly concentrated, in all cases, with a concentration ratio greater than 0.34, and particularly in the AvDD portfolio with a ratio of 0.76.

In Figure 2, we show the poor performance of the Ibex 35 index compared to the other six methodologies

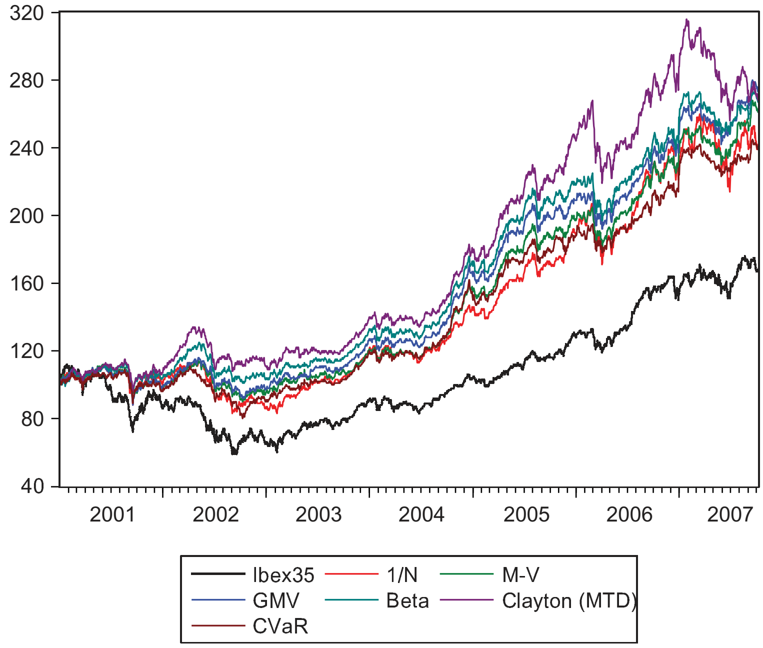

Figure 2. Accumulated wealth, 2001–2007 time period.

under evaluation (the five portfolios with a higher Sharpe ratio and the naive strategy of 1/N). As can be seen, the differences between the Ibex 35 index and the portfolios began from the middle of 2001. From 2001 to 2007, the Ibex35 index achieved a total return of just over 67%. Meanwhile, the GMV and MV strategies had a total return greater than 160%. Even the strategy of 1/N obtained double return (139.01%) than the Ibex 35 index.

The strategy of 1/N provides a good out-of-sample performance, especially when it is compared with the Ibex 35 index. However, the 1/N portfolio is clearly exceeded by other strategies, not only on return but also on a higher Sharpe ratio, a lower VaR and CVaR, and greater diversification ratio. In short, there are five portfolios that completely dominate, except in concentration ratio, the naive strategy of 1/N, among which the GMV, MV and Beta portfolios stand out.

Base 100 in February 1st 2001. We represent the accumulated wealth of an investor who invested 100 currency units on February 1st 2001. We include the Ibex 35 index, the 1/N portfolio, the M-V portfolio, the GMV portfolio, the Beta portfolio, the Clayton (MTD) portfolio and the CVaR portfolio.

In conclusion, the weak out-of-simple performance of the 1/N strategy in the time period 2001–2014 contrasts with the good performance of this portfolio in the time period 2001–2007; this behaviour suggests that the 1/N strategy has been quite poor during the Global Financial Crisis and European Sovereign Debt Crisis.

Out-of-sample performance: 2008–2014

Four portfolios achieve an annual return higher than 5% in the 2008–2014 period, providing the O-Minimax portfolio the greatest return, with an annual return of around 11%. It is an exceptional case since the rest of strategies are unable to overcome such threshold of 5% by year. The return obtained is well below that achieved in the previous period, where three portfolios rendered annualized returns above 15%. The Ibex 35 index and the 1/N portfolio are in the opposite direction, with an annual return drop of 5.26% and 6.73%, respectively. Taking this into consideration, the relative performance of other strategies is not as poor as it might seem *a priori*.

If we consider the total return for the period, the O-Minimax portfolio obtains a return of 100%, followed for the AVDD portfolio with a 58.99%, the MinCDaR95 portfolio with 58.76% and the MV portfolio with a 44.44% of total return. Meanwhile, on the opposite side the 1/N portfolio stands out with a total return of −38.58% and the Ibex 35 index with a total return of −31.48%. So the strategy of 1/N obtained negative returns even higher than those obtained by the Ibex 35 index.

Regarding the Value at Risk and Conditional Value at Risk associate with each strategy, it is again the GMV portfolio which has a lower VaR and CVaR with 2.01 and 2.53, respectively. The GMV portfolio is followed by the Beta, MDP, MV and CVaR portfolios, in no case, with a VaR and CVaR higher than 2.2 and 2.8. These are good results if we compare them with the Ibex 35 index (2.879 and 3.256) and the 1/N strategy (2.599 and 3.256). In this regard, 11 out of the 14 portfolios have a lower VaR and CVaR with respect to the Ibex 35 index and the 1/N strategy.

All strategies, except the 1/N portfolio (−0266), obtained Sharpe ratios higher than that for the Ibex 35 index. However, they are only three strategies that statistically exceed the Sharpe ratio of the Ibex 35 index; this is because the covariance between the portfolio and the index is very high. The O-Minimax portfolio has the highest Sharpe ratio (0.452), and the difference from the Ibex 35 index is moderately significant. Regarding the other two portfolios: the MinCDaR95 (0.315) and the MV (0.255) portfolios, both present a relatively high Sharpe ratio, although in both cases the difference is weakly significant.

The MDP, Clayton (MTD) and MTD strategies have the highest diversification ratios, the MDP with a remarkable ratio of 1.53, nevertheless somewhat lower than the value of the 2001–2007 time period, highlighting the highest correlation between asset returns in the portfolio during the 2008–2014 period. Again, the AvDD, CDaR95 and MaxDD portfolios have the lowest diversification ratio. In total, there are seven strategies that exceed the diversification ratio of the 1/N portfolio (1.42), including the Beta (1,499), MV (1,457) and GMV(1.454) portfolios.

The ERC and 1/N portfolios have the lowest concentration ratios, with 0.036 and 0.038, respectively. The concentration ratio is slightly lower than that in the 2001–2007 time period due to an increase in the assets number (see Table 1). In contrast, the AVDD, O-Minimax and CDaR95 strategies present a higher concentration ratio, in all cases with a concentration ratio greater than 0.58. This increase can be explained by the higher correlation between asset returns. This fact is widely investigated in recent papers as Moldovan (2011), for the New York, London and Tokyo index; and in Ahmad et al. (2013), for the contagion between financial markets.

In Figure 3, we show the poor performance of the Ibex 35 index and the 1/N portfolio compared to the

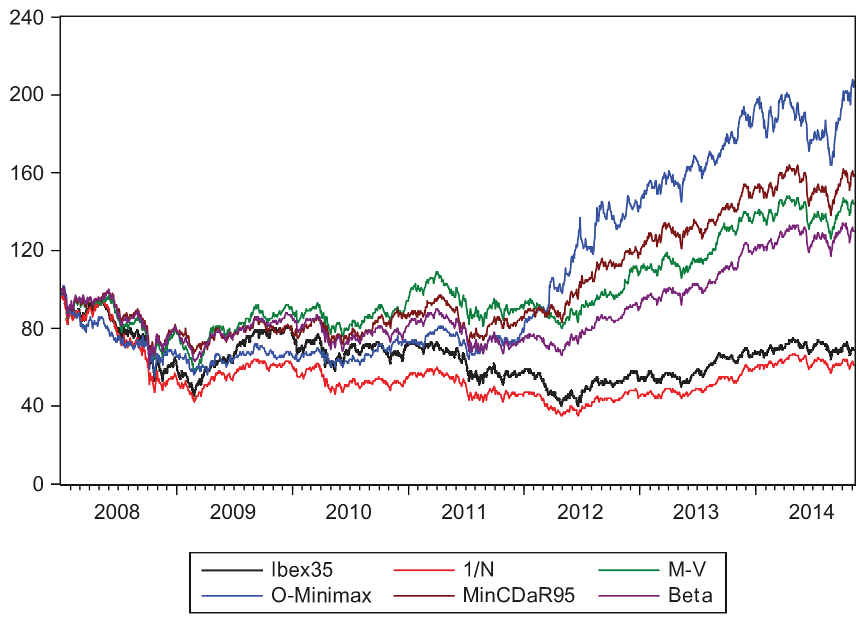

Figure 3. Accumulated wealth, 2008–2014 time period.

Accumulated wealth in the 2008–2014 time period. Base 100 in January 2nd 2008. We represent the accumulated wealth of an investor who invested 100 currency units on January 2nd 2008. We include the Ibex 35 index, the 1/N portfolio, the M-V portfolio, the O-Minimax portfolio, the MinCDaR95 portfolio, and the Beta portfolio.

other four methodologies under scrutiny (three portfolios with Sharpe ratio significantly different to the Ibex 35 index and the Beta portfolio, which are almost significant).

The differences between the 1/N portfolio and the rest of portfolios began from 2008. The 1/N portfolio performance is worse than the Ibex 35 index, in terms of total return (−38.58%). Meanwhile and during this period, the Ibex 35 index performance has been quite poor, with a total return of −31.48% and annualized Sharpe ratio of −0.187; in contrast with the performance of the O-Minimax, the MinCDaR95, the MV and the Beta portfolios, the O-Minimax standing out with a total return of 106.98% and annualized Sharpe ratio of 0.452.

The 1/N portfolio performance before and during the Global Financial Crisis and European sovereign debt crisis indicates that this strategy has a good behaviour when the market trend is bullish and *vice versa* when it is bearish. The increase in the correlation between assets has adversely affected the 1/N portfolio performance during the period 2008–2014.

V. Concluding remarks

In this article, we have examined 15 asset allocation models in the main Spanish stock market using the Ibex 35 index. We have compared the total returns, Sharpe ratios, VaR and CVaR, and the diversification and concentration ratios of each portfolio strategy. We have analysed the performance for the daily returns over a sample of 14 years, divided into two sub-samples of seven years each one, whose purpose is to test the robustness of the results in periods of high and low correlations between assets and with a market characterized by many bullish and bearish trends.

We have found that the Sharpe ratio of the mean-variance (MV) and the minimum variance (GMV) strategies are higher compared to the naive strategy of 1/N and the Ibex 35 index, in the 2001–2014 period. All models achieved a Sharpe ratio greater than the Ibex 35 index during the 2001–2014 period, although only nine strategies are statistically different from it.

Regarding the total return for the 2001–2014 time period, the MinCDaR95 portfolio is found to deliver higher returns, followed for the mean-variance, the Minimax optimization based on risk/return ratio, the low beta and the minimum variance strategies. All these gave returns five times greater than those derived from the naive strategy of 1/N.

The performance of the naive strategy of 1/N is found not to be much different from other strategies in the 2001–2007 period, although it is surpassed by five models, except in concentration ratio, among which are the mean-variance (M-V) and the minimum variance (GMV) portfolios.

We observed that the 1/N strategy performance is worse that the Ibex 35 index in the 2008–2014 period. It is from 2008 when we detected divergences between the naive strategy of 1/N and the other strategies. Our findings suggest that the 1/N portfolio seems to show the worst performance during the Global Financial Crisis and the European sovereign debt crisis (that is, a time period characterized by a higher correlation between financial assets and downtrends in the markets). Furthermore, except the O-Minimax portfolio, other strategies are found to outperform the naive strategy of 1/N in a lower VaR and CVaR, and a higher diversification ratio. However, we found that the 1/N portfolio has a lower degree of concentration, although it is to be expected since it includes all the assets that make up the Ibex 35 index. A large number of strategies have been found to produce a better performance than the Ibex 35 index and the naive strategy of 1/N. We have shown that most of strategies outperform better than both the Ibex35 index and the 1/N strategy, various portfolio strategies achieving higher return, greater Sharpe ratio, greater diversification ratio and lower VaR and CVaR than those associated with the naive strategy of 1/N and the Ibex 35 index.

In addition, our empirical results indicate that there are several strategies that do not depend on the expected assets return to assign weights (such as the GMV, the ERC, the MDP and the MTD strategies) that are also able to overcome the naive strategy of 1/N. Nevertheless, the Markowitz mean-variance portfolio with short-selling constraint is found to be the only strategy that achieves a Sharpe ratio statistically different to the Ibex 35 in the two time periods analysed (2001–2007 and 2008–2014). In view of the encouraging results of this article, we suggest that the mean-variance, minimum-variance and conditional draw-down at risk

(95%) portfolios could be used, at least as a first reference, when analysing the behaviour of the main Spanish stock market.

All in all, the results of our analysis are not consistent with those presented in DeMiguel, Garlappi, and Uppal (2009) and Allen et al. (2014a); although these are in line with those of Kirby and Ostdiek (2012) and Allen et al. (2014b) for the hedge fund indices. Thus, although in all empirical works the results obtained have to be taken with some degree of caution (since they are based on a particular index over a certain time period), our findings lead us to infer that the naive strategy of 1/N can provide good results during some episodes, being always exceeded by several portfolio optimization models.

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Appendix

In this appendix, we include two tables: Table A1 presents six descriptive statistic that provide information about the behaviour of the Ibex 35 index and the 15 main models consider in this article. Table A2 offers the assets name that we have considered for the portfolio construction.

Table A1. Summary statistics during different time periods.

Portfolios	Time Period	Min	1st quartile	Median	Mean	3st quartile	Max
Ibex 35	2001–2014	−9.1408	−0.7367	0.0694	0.0151	0.7663	14.4349
	2001–2007	−5.8171	−0.6104	0.0866	0.0371	0.6838	5.9599
	2008–2014	−9.1408	−0.8937	0.0320	−0.0059	0.8756	14.4349
1/N	2001–2014	−8.1174	−0.6218	0.0644	0.0194	0.7184	10.8051
	2001–2007	−5.3981	−0.4502	0.0900	0.0543	0.6279	4.0264
	2008–2014	−8.1174	−0.8157	0.0389	−0.0149	0.8254	10.8051
M-V	2001–2014	−7.1356	−0.5125	0.0815	0.0433	0.6068	10.5063
	2001–2007	−3.6690	−0.3683	0.0802	0.0577	0.5077	5.9757
	2008–2014	−7.1356	−0.6975	0.0858	0.0291	0.7327	10.5063
GMV	2001–2014	−6.9594	−0.4884	0.0638	0.0402	0.5883	10.0633
	2001–2007	−3.9029	−0.3608	0.0869	0.0601	0.4840	6.1715
	2008–2014	−6.9594	−0.6285	0.0485	0.0206	0.7007	10.0633
MDP	2001–2014	−7.4434	−0.4782	0.0709	0.0346	0.6107	10.8855
	2001–2007	−4.1242	−0.3760	0.0780	0.0498	0.5113	6.6558
	2008–2014	−7.4434	−0.6320	0.0683	0.0197	0.7065	10.8855
ERC	2001–2014	−7.7076	−0.5593	0.0663	0.0241	0.6684	10.5324
	2001–2007	−4.9052	−0.3944	0.0913	0.0545	0.5822	3.7957
	2008–2014	−7.7076	−0.7731	0.0366	−0.0059	0.7634	10.5324
MTD	2001–2014	−6.7086	−0.5082	0.0715	0.0282	0.6168	10.9713
	2001–2007	−4.5205	−0.39020	0.0675	0.0509	0.5473	5.7833
	2008–2014	−6.7086	−0.6882	0.0798	0.0058	0.7177	10.9713
CVaR	2001–2014	−7.1332	−0.5122	0.0424	0.0323	0.6066	11.5357
	2001–2007	−4.4808	−0.3877	0.0490	0.0528	0.5217	6.6697
	2008–2014	−7.1332	−0.7053	0.0235	0.0121	0.7347	11.5357
MaxDD	2001–2014	−11.5167	−0.6781	0.0263	0.0373	0.7447	13.7449
	2001–2007	−6.0443	−0.5381	0.0279	0.0515	0.6501	5.7198
	2008–2014	−11.5167	−0.8617	0.0175	0.0233	0.8961	13.7449
AvDD	2001–2014	−13.0517	−0.8659	0.0007	0.0489	0.9324	16.7182
	2001–2007	−9.2679	−0.7592	0.0284	0.0496	0.8220	12.6075
	2008–2014	−13.0517	−0.9841	0.0000	0.0483	1.07235	16.7182
CDaR95	2001–2014	−10.7980	−0.7045	0.0225	0.0387	0.7942	12.9975
	2001–2007	−6.1414	−0.5546	0.0485	0.0542	0.6864	5.2095
	2008–2014	−10.7980	−0.9299	0.0000	0.0235	0.9433	12.9975
MinCDaR95	2001–2014	−7.3623	−0.5445	0.0339	0.0455	0.6411	10.5430
	2001–2007	−5.0936	−0.4068	0.0368	0.0564	0.5224	10.5430
	2008–2014	−7.3623	−0.6770	0.0269	0.0348	0.7544	10.3252
R-Minimax	2001–2014	−7.0666	−0.5786	0.0268	0.0278	0.6569	11.2922
	2001–2007	−4.3382	−0.4408	0.0303	0.0508	0.5370	4.35426
	2008–2014	−7.0666	0.754	0.0233	0.0050	0.7896	11.2922
O-Minimax	2001–2014	−8.8971	−0.6851	0.0576	0.0441	0.7309	11.3846
	2001–2007	−6.4801	−0.5924	0.0597	0.0361	0.6588	7.8800
	2008–2014	−8.8971	−0.7579	0.0368	0.0520	0.8326	11.3846
Clayton (MTD)	2001–2014	−7.2840	−0.5064	0.0764	0.0360	0.6483	10.9520
	2001–2007	−4.5469	−0.3797	0.0895	0.0598	0.5767	4.9972
	2008–2014	7.2840	−0.6836	0.0567	0.0124	0.7741	10.9520
Beta	2001–2014	−7.0623	−0.4720	0.0619	0.0405	0.6003	11.2040
	2001–2007	−4.5521	−0.3687	0.0671	0.0589	0.5053	6.3157
	2008–2014	−7.0623	−0.6338	0.0531	0.0224	0.6727	11.2040

Table A2. Assets for the portfolio construction by time period.

Time period	Nº of risky assets	Asset names
03/01/2000 28/12/2001	21	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, Sogecable, Altadis.
02/01/2001 30/12/2002	23	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, Sogecable, Altadis, Zeltia, REE.
02/01/2002 30/12/2003	25	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, Sogecable, Altadis, Zeltia, REE, Gamesa, Inditex.
02/01/2003 30/12/2004	26	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, Sogecable, Altadis, Zeltia, REE, Gamesa, Inditex, Enagas.
02/01/2004 30/12/2005	29	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, Sogecable, Altadis, Zeltia, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Banesto, Prisa.
03/01/2005 29/12/2006	29	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, Sogecable, Altadis, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Banesto, Prisa, Atresmedia, Mediaset.
02/01/2006 28/12/2007	30	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, Sogecable, Altadis, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Banesto, Prisa, Atresmedia, Mediaset, Banesto.
02/01/2007 30/12/2008	29	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Mediaset, Banesto, BME, Grifols, Abengoa.
02/01/2008 30/12/2009	30	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Mediaset, Banesto, BME, Grifols, Abengoa, Técnicas Reunidas.
02/01/2009 30/12/2010	31	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Mediaset, Banesto, BME, Grifols, Abengoa, Técnicas Reunidas, Arcelormittal.
04/01/2010 30/12/2011	31	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Mediaset, Banesto, BME, Grifols, Abengoa, Técnicas Reunidas, Arcelormittal.
03/01/2011 31/12/2012	32	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, Ebro Foods, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Mediaset, BME, Grifols, Abengoa, Técnicas Reunidas, Arcelormittal, Amadeus, Caixabank.
02/01/2012 31/12/2013	33	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, Endesa, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Mediaset, BME, Grifols, Técnicas Reunidas, Arcelormittal, Amadeus, Caixabank, DIA, Bankia, IAG.
02/01/2013 31/12/2014	35	Abertis, Acciona, ACS, Banco Popular, Bankinter, BBVA, FCC, Gas Natural, Iberdrola, Mapfre, OHL, Repsol, Sacyr, Santander, Telefónica, Ferrovial, Indra, REE, Gamesa, Inditex, Enagas, Banco Sabadell, Mediaset, BME, Grifols, Técnicas Reunidas, Arcelormittal, Amadeus, Caixabank, DIA, Bankia, IAG, Viscofan, Jazztel, Ebro Foods.