ABC methods for PH distributions with applications in insurance risk problems.

Concepción Ausín, Department of Statistics, Universidad Carlos III de Madrid, Spain.
Pedro Galeano, Department of Statistics, Universidad Carlos III de Madrid, Spain.
Simon Wilson, School of Computer Science and Statistics, Trinity College Dublin, Irleland.

Abstract

Phase type (PH) distributions are a very flexible class of models to describe loss distributions in insurance risk. This family of distributions is able to capture frequently observed patterns in this context such as strong skewness, heavy tails, multimodality and extreme values. It includes the exponential, Erlang, mixtures of exponentials and Coxian distributions, as particular cases. Roughly speaking, a positive random variable is PH if each realization can be expressed as a sum of exponential random variables.

Given an observed loss data set, it is not straightforward how to approximate it using a PH distribution. Many different approaches have been developed in the literature. Classical procedures are usually based on the method of moments and maximum likelihood estimation. However, using these approaches, it is not easy how to derive confidence intervals for quantities of interest depending on the estimated distributions such as the ruin probability in a risk reserve process with PH claim sizes. Alternatively, this can be done in a natural way from the Bayesian perspective. In this context, Markov Chain Monte Carlo (MCMC) methods have been developed in Bladt et al. (2003) and Aslett (2012). Also, MCMC methods for this type of models have been considered by Ausin et al. (2004, 2008) in a queueing context.

Classical and Bayesian methods for PH distributions are usually based on sophisticated computational algorithms which are very time consuming. This is mainly due to the difficulties in the evaluation of the likelihood. In order to solve these problems, our proposal is to make use of the recently developed Approximate Bayesian Computation (ABC) methods, see e.g. Marin et al. (2012). ABC methods provide the advantage of avoiding the evaluation of

the likelihood and are mainly based on simulation. We believe that the ABC methodology can provide a potential alternative to MCMC for PH distributions since these models are very easy to simulate but their likelihood is usually difficult and/or computationally expensive to evaluate.

References

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